Grade 4 Mathematics, Quarter 1, Unit 1.1

Place Value for Multidigit Whole Numbers to 1,000,000

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Compare and order whole numbers equal to or less than 1,000,000.
- Use appropriate symbols to compare numbers (<, >, =).
- Round numbers to a given place (up to 1,000,000).
- Read and write numbers in standard and expanded form.
- Read and write number names.
- Understand relationship of numbers in base ten system.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use concrete objects or pictures to help conceptualize.
- Ask questions to ensure responses make sense.
- Explain relationships between numbers.

Model with mathematics.

- Make approximations to simplify a complicated situation, realizing they may need revision.
- Map the relationship of numbers using two-way tables, graphs, flowcharts, and formulas.

Use appropriate tools strategically.

- Make decisions about when (and which) tools would be helpful.
- Concrete models or drawings.
- Use technological tools to explore and deepen understanding of concepts.

Essential questions

- What do you know about place value?
- Using models, representations, and explanations, how can you show a whole number (e.g., 25 or 751) in different ways?
- What strategies do you use to compare numbers?
- What strategies do you use to order numbers?
- What is your strategy for rounding numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2]	**4.NBT**

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In second grade, students read and wrote numbers to 1,000 using base ten numerals, number names, and expanded form. They also compared two- and three-digit numbers.

In third grade, students used place value to round numbers to 10 or 100.

Current Learning

Students read, write, and round whole numbers less than or equal to 1,000,000. Students compare two multidigit numbers based on meanings of digits using >, <, and =. In a later unit of study, students learn the place value for decimals.

Future Learning

In fifth grade, students will recognize patterns in numbers and decimals of zeros when x equals powers of ten. They will use exponents and read, write, and compare decimals to thousands.

In sixth grade, students will fluently use decimals to add, subtract, multiply, and divide.

Additional Findings

Research from *Principles and Standards for School Mathematics* states that instructional programs in prekindergarten through grade 12 should enable students to work with numbers. Students should understand ways of representing numbers, relationships among numbers, and number systems. They should also understand meanings of operations and how they relate to one another. Furthermore, students should be able to compute fluently and make reasonable estimates. Through problem solving, students can explore and solidify their understandings of number. Research has shown that learning about number and operation is a complex process for children (p. 32).

Grade 4 Mathematics, Quarter 1, Unit 1.2

Exploring the Properties of Addition and Subtraction of Whole Numbers to 1,000,000

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Generate a number pattern that follows a given rule.
- Identify additional features of the pattern that result from applying a given rule.
- Solve multistep problems posed with whole numbers that have a whole number answer.
- Represent problems using an equation with a letter (variable) standing for the unknown quantity.
- Use estimation and mental calculations to determine reasonableness of answers.
- Add and subtract whole numbers using the standard algorithm.
- Solve word problems using addition and subtraction, involving distances, time, and money.
- Solve problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Take an abstract situation and represent it symbolically.
- Attend to the meaning of quantities.
- Flexibly use different properties of operations.

Construct viable arguments and critique the reasoning of others.

- Analyze situations by determining rules.
- Construct arguments using concrete information such as drawings, diagrams, objects, and actions.

Look for and express regularity in repeated reasoning.

- Look for general methods and shortcuts.
- Evaluate the reasonableness of results.

Essential questions

- How do you know which operation to use to solve this problem?
- How can estimation help you prove that your answer is reasonable?
- What is your strategy for adding/subtracting whole numbers? Explain your thinking.
- What strategy would you use to mentally add or subtract numbers?
- Given a rule, how can you generate a pattern?
- How do you solve problems involving distance, time and money?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**4.OA**

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers ~~using the four operations, including problems in which remainders must be interpreted~~. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2]	**4.NBT**

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4 ~~Fluently~~ add and subtract multi-digit whole numbers using the standard algorithm.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use ~~the four~~ operations to solve word problems involving distances, intervals of time~~, liquid volumes, masses of objects,~~ and money~~, including problems involving simple fractions or decimals,~~ and problems that require expressing measurements given in a larger unit in terms of a smaller unit~~. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale~~.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students mentally added and subtracted within 20. They also fluently added and subtracted numbers within 100, increasing up to four two-digit numbers using strategies based on place value and properties of operations. In addition, they added and subtracted within 1,000 using concrete models and drawings. Students learned to decompose or compose tens and hundreds. Finally, they explained why addition and subtraction strategies work. They have worked with measurement, time, and money as well.

In grade 3, students solved two-step word problems using four operations. They learned to represent problems using equations with a letter representing the unknown and assessed the reasonableness of answers using mental computation and estimation.

Current Learning

In grade four, students generate number patterns to follow a rule. They identify features of patterns that are not stated explicitly. The standard does not require students to infer the underlying rule for a pattern, but rather to generate the pattern from a given rule.

It is a fourth grade expectation that students add and subtract multidigit whole numbers using the standard algorithm fluently. Students should recognize that a general algorithm for adding/subtracting numbers involves computing like base-ten units and composing/decomposing like base-ten units as needed (i.e., regrouping). Students add and subtract multidigit numbers and solve multistep word problems with whole numbers using addition and subtraction involving money, distance, time, and measurement.

They use equations with a letter to represent an unknown value. Students determine the reasonableness of answers using mental computation and estimation strategies. Solving problems using measurement is a second priority according to the PARCC document.

Future Learning

In fifth grade, students will be generating two numerical patterns using two given rules. They will identify apparent relationships between corresponding terms, forming ordered pairs consisting of corresponding terms, and graphing the ordered pairs on a coordinate plane. Problem solving also involves the use of relating volume to the operations of multiplication and addition. Students will also use place value to round numbers to any place.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics suggests that the development of computational fluency and the attainment of problem-solving skills are intertwined, as both develop with understanding (p. 68).

According to *Principles and Standards for School Mathematics,* when students leave grade 5, they should be able to solve problems involving whole-number computation and should recognize that each operation will help them solve many different types of problems. Students should be able to mentally estimate a reasonable result, to efficiently recall or derive the basic number combinations for each operation, and to compute fluently with multi-digit whole numbers (p. 149).

Beyond Numeracy states that there should be units designed to help students learn to decide which operation or sequence of operations is called for in a given problem (p. 53).

Principles and Standards for School Mathematics states that students investigate numerical and geometric patterns and express them mathematically in words or symbols. They should analyze the structure of the pattern and how it grows or changes; organize this information systematically; and use their analysis to develop generalizations about the mathematical relationships in the pattern (p. 159).

Students can find patterns in all of the decompositions of a given number and eventually summarize these patterns for several numbers (*Progressions for the Common Core State Standards: Counting and Cardinality; K-5, Operations and Algebraic Thinking,* p. 10).

The standards distinguish strategies from algorithms. For example, students use strategies for addition and subtraction in grades K–3, but are expected to fluently add and subtract whole numbers using standard algorithms by the end of grade four. Use of the standard algorithm can be viewed as the culmination of a long progression of reasoning about quantities, the base-ten system, and the properties of operations (*Progressions for the Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten,* p. 3).

Grade 4 Mathematics, Quarter 1, Unit 1.3

Understanding Patterns in Multiplication and Division

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Generate a number pattern that follows a given rule.
- Identify features of a pattern not stated in the rule.
- Find factor pairs in the range 1–100.
- Understand a whole number is a multiple of each of its factors.
- Determine whether a whole number is a multiple of a given one-digit number.
- Determine whether a given number is prime or composite (numbers 1–100).
- Interpret a multiplication equation as a verbal comparison statement.
- Interpret a verbal comparison statement as a multiplication equation.
- Solve word problems involving multiplicative comparisons.

Mathematical practices to be integrated

Model with mathematics.

- Attend to the meaning of quantities.
- Flexibly use different properties of operations.

Look for and make use of structure.

- Look closely to determine a pattern or structure.
- Step back for an overview and shift perspective when solving problems.

Look for and express regularity in repeated reasoning.

- Look for repeated calculations and patterns.
- Accurate calculations.

Essential questions

- What are some different comparison statements you can make about a set of numbers (e.g., 6 and 18)? How would you record your comparison statements as equations?
- What are the factors of a number and how do you find them?
- What are the multiples of a number and how do you find them?
- How do you know you have found all the possible factors for a given number? How do you know you have found all the multiples of a given number?
- How do you determine if a number is prime or composite?
- How are multiplication and division related? How are they different?
- Given a rule, how do you generate a pattern?
- What patterns do you notice?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 4.OA

Generate and analyze patterns.

4.OA.5 Generate a number ~~or shape~~ pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply ~~or divide~~ to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

[1] See Glossary, Table 2.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In third grade, students learned how to identify patterns (in the addition and multiplication table), explaining them using properties of operations. Students determined the unknown whole number in multiplication and division. They also gained an understanding of division as an unknown factor problem. They fluently multiplied and divided numbers within 100. Students interpreted products of whole numbers as the total number of objects in a group.

Current Learning

In grade 4, multidigit multiplication is one of the three critical areas. Using the four operations to solve problems is a top priority according to PARCC. Students generate a number pattern that follows a given rule, at the developmental level. They identify the features of the pattern that were not explicitly given (e.g., given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.)

Students find the factor pairs for whole numbers in the range 1–100, also determining which are prime and composite, beginning at the developmental level continuing on to reinforcement. Students also interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5) at the developmental level; and represent a verbal comparison as a multiplication equation. This unit is reinforcing student understanding of the relationship between multiplication and division while concurrently reinforcing their ability to multiply and divide fluently.

Future Learning

In fifth grade, students will be generating two numerical patterns using two given rules. They will identify apparent relationships between corresponding terms, forming ordered pairs consisting of corresponding terms, and graphing the ordered pairs on a coordinate plane. Students will interpret multiplication as scaling or resizing (comparing the size of a product to the size of one factor on the basis of the size of the other without performing multiplication).

In sixth grade, students will find the greatest common factors of two whole numbers greater than or equal to 100. Additionally, they will find the least common multiple of two whole numbers less than or equal to 12.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, finding and using patterns greatly simplifies the task of learning multiplication combinations. Moreover, it is one of the very essences of mathematics (p. 77).

Principles and Standards for School Mathematics states that, in grades 3–5, students should focus on the meaning of the relationship between multiplication and division. It is important that students understand what each number in a multiplication or division expression represents. Students should consider and discuss different types of problems that can be solved using multiplication and division (p. 151).

The book further states that students can extend their understanding of multiplication and division as they consider the inverse relationship between the two operations. Further meaning for multiplication should develop as students build and describe area models showing how a product is related to its factors (p. 152).

Principles and Standards of Mathematics also states that, by the end of grades 3–5, students should be computing fluently with whole numbers. Students exhibit computational fluency when they demonstrate flexibility in the conceptual methods they choose, when they understand and can explain these methods, and can produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base ten system, properties of multiplication and division, and number relationships (p. 152).

Grade 4 Mathematics, Quarter 1, Unit 1.4

Understanding Fractions by Ordering and Comparing

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Explain why fraction *a/b* is equivalent to a fraction (*n* x *a*)/(*n* x *b*) using visual fraction models.
- Explain how number and size of parts differ even though fractions are equivalent.
- Recognize and generate equivalent fractions.
- Compare two fractions with different numerators and denominators.
- Record results of comparisons with symbols (<, >, =).
- Express fraction equivalents with tens and hundreds.

Mathematical practices to be integrated

Model with mathematics.

- Draw conclusions, interpret results, and revise models if needed.
- Simplify the situation to a simpler rule allowing for revisions.

Construct viable arguments and critique the reasoning of others.

- State justification.
- Questions to clarify argument.
- Communicate thoughts to others.

Essential questions

- What are equivalent fractions? How do you find them?
- How can you determine if two fractions are equivalent?
- How can you use your knowledge of equivalent fractions to compare two fractions that are not equivalent?
- Given two fractions with different numerators and/or denominators, what is the strategy you would use to make them equivalent?
- What do <, >, = mean? How can you use them to compare two fractions that are not equivalent?
- How can you use knowledge of place value (tens and hundreds) to generate equivalent fractions?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3]	**4.NF**

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In third grade, understanding fractions was a developing area. Students learned that two fractions are equivalent if they are the same size and the same point on a number line. They recognized and generated simple equivalent fractions, and used a visual model to explain why they are equivalent. Students wrote whole numbers as fractions and recognized they were equivalent to a whole. They used visual models to compare fractions with the same numerator or denominator by looking at their size. Students learned that comparisons are only valid when two fractions refer to some whole (e.g., same area, same set). Comparisons were recorded with the symbols >, <, =, and conclusions were justified (e.g., visual model).

Current Learning

In grade 4, fractions are one of the three critical areas and are a first priority of instruction according to the PARCC documents. At a developmental level, students explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models. They pay attention to how the number and size of the parts differ even though the two fractions are the same size. Students compare fractions with different numerators and denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.) They recognize that comparisons are valid only when the two fractions refer to the same whole. Students record the results of the comparisons using symbols (=, <, >). They justify their conclusions by using a visual of a fraction model. Fourth grade is the first place that students will be working with fractions of a set.

Students also express a fraction with a denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100, at the developmental level. An example of this would be expressing 3/10 as 30/100 and adding 3/10 + 4/100 = 34/100. It is important to note that students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. However, addition and subtraction with unlike denominators in general is not a requirement in fourth grade. Later in fourth grade, students will be exposed to the application of multiplying fractions to whole numbers and they will be further developing a relationship between fractions and decimals.

Future Learning

The three critical areas in fifth grade are: Developing fluency with addition and subtraction of fractions; developing understanding of multiplication of fractions; and developing understanding of division of fractions (unit of fractions divided by whole numbers and whole numbers divided by unit fractions).

Students will solve word problems involving addition and subtraction with unlike denominators. They will use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. They will extend their previous understanding of multiplication and division to multiply and divide fractions.

Additional Findings

According to *Principles and Standards for School Mathematics*, during grades 3–5, students should build their understanding of fractions as parts of a whole and as division. They will need to see and explore a variety of models of fractions focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions. They should develop strategies for ordering and comparing fractions often using benchmarks such as 1/2 and 1 (p. 150).

The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating their sums can be different. Just as the sum of 4 and 7 can be seen as the length of the segment obtained by joining together two segments of lengths 4 and 7, so the sum of 2/3 and 8/5. It is not necessary to know how much 2/3+8/5 is exactly in order to know what the sum means. This is analogous to understanding 51x78 as 51 groups of 78, without necessarily knowing its exact value (*Progressions for the Common Core State Standards in Mathematics,* 3–5 Number and Operation—Fractions, p. 6)